

# Validation and Sensitivity Analysis of a Non-Linear Fiber Model to Describe Soft Tissue Mechanics

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## Introduction

Planar tissues such as the iliofemoral ligament may be modeled as composites in order to understand their mechanical behavior. A non-linear fiber model [1] has been used to characterize other fibrous, soft tissues [2]. It is critical, however, to validate the model for future tissue characterization. An optimized, well-validated scheme is particularly important when limited data are available, as may be due to simple geometry or as may occur during a test to failure.

Multiple forms of validation exist, including theoretical, internal, and external [3]. Theoretical validation ensures the theory underlying the model is sound, while internal validation ensures that the model equations are solved without error. These forms are satisfied during model and code development. External validation ensures that the model accurately represents real world data. The objective of this work is to validate externally the non-linear fiber model through parameter sensitivity analysis and characterization of the iliofemoral ligament.

**Model Theory:** The non-linear fiber model is described by a set of closed-form constitutive equations, allowing for rapid computation [1]. Fibers are drawn from a bidirectional von Mises distribution of fibers (1), and each fiber is described by an exponential function of the Green Strain (2), where  $\kappa$  denotes the degree of anisotropy,  $\mu$  the mean fiber alignment, A the fiber stiffness, and B the fiber non-linearity.

$$f(\theta; \kappa, \mu) = \frac{1}{\pi I_0(\kappa)} \exp(\kappa \cos[2(\theta - \mu)]), \quad \theta \in [0, \pi] \quad (1)$$

$$S_f = A \left\{ \exp(B(\lambda_f^2 - 1)) - 1 \right\} \quad (2)$$

## Materials and Methods

Human, cadaveric iliofemoral ligament was dissected and tested equibiaxially at 7% grip strain in a saline bath using an Instron-Sacks biaxial tester, shown in Figure 1. Sample geometry, grip force, and tracked surface displacement were utilized in the model.

To model the experiment, sample geometry was used to create a finite element (FE) mesh of the sample. An initial guess of the parameters ( $\kappa$ ,  $\mu$ , A, B) was made, and the modeled grip force and surface displacements were calculated from the FE model. The difference between the modeled and experimental arm forces and the difference between the nodal surface displacements were minimized using `fminsearch()` function in Matlab (Natick, MA).



Figure 1. Iliofemoral ligament in Instron-Sacks Biaxial Tester

To evaluate parameter sensitivity, previously fitted model parameters were used [2]. From this set, each parameter was varied  $\pm 15\%$  in increments of 5%, keeping other parameters unvaried. Then, the model was run to produce grip forces and nodal surface displacements, for each varied parameter sets. The displacement at the center of the sample and the grip force at 6% grip stretch were used as metrics, and the error was the difference of the unvaried model and the varied model, normalized by the unvaried model.

The iliofemoral ligament was characterized using three methods of varying the operating function: 1) minimizing only grip force errors (FORCE), 2) minimizing only nodal surface displacement errors (DISP), and 3) minimizing both grip force and nodal surface displacement errors (ALL). Confidence intervals of the parameters fitted with grip force and nodal surface displacement were also calculated using a linearized form of the model with simplified confidence regions [4].

## Results

The sensitivities (rate of change of error as the parameter was perturbed  $\pm 15\%$   $dError/dParam$ ) for each parameter and metric are shown in Figure 2. Grip force is more sensitive to A and B, while nodal surface displacement is more sensitive to  $\kappa$  and  $\mu$ .

Table 1 displays the parameter set and sum of squared errors (SSE) for each method. FORCE provides a good fit of the grip forces but a poor fit of the surface displacement. DISP provides a poor fit of the grip forces but a good fit of the surface displacement. ALL provides a good fit for grip forces and surface displacements. The parameter set for each method is very different. ALL has the smallest SSE for both force and

displacement error. FORCE has a smaller SSE(force) than DISP, and DISP has a smaller SSE(displacement) than FORCE.

Table 2 shows the 95% confidence intervals normalized by each parameter for parameter set for each method. Comparing the confidence intervals, ALL has the most narrow intervals.

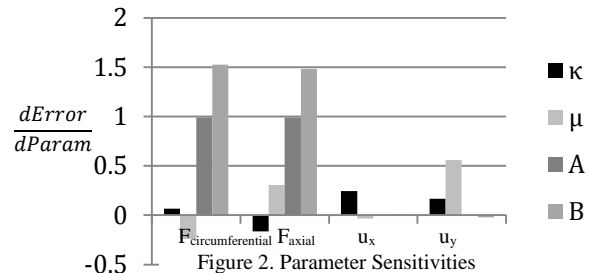


Table 1. Parameter Sets

Method	$\kappa$	$\mu$ ( $^\circ$ )	A (N/mm)	B	SSE (Forces)	SSE (Disp)
FORCE	2	60	0.2130	79.62	0.0370	0.4380
DISP	0.43	89.63	5.0280	7.62	0.0938	0.4311
ALL	<b>0.43</b>	<b>89.63</b>	<b>0.2130</b>	<b>79.62</b>	<b>0.0307</b>	<b>0.4076</b>

Table 2. 95% Normalized Confidence Interval Lengths

Method	$CI_\kappa$	$CI_\mu$	$CI_A$	$CI_B$
FORCE	3.2895	0.3932	0.2640	0.1442
DISP	1.0854	0	$\infty$	$\infty$
ALL	<b>0.2424</b>	<b>0.3093</b>	<b>0.0287</b>	<b>0.0161</b>

## Discussion and Conclusion

The non-linear fiber model was validated externally by determining parameter sensitivity and characterizing the iliofemoral ligament. Varying  $\kappa$  and  $\mu$  had a small effect on the calculated grip force, but they have more influence over the displacement. A and B have a stronger influence on the grip force than the displacement, which was expected considering that they govern fiber stiffness and non-linearity. This result suggests that both grip forces and nodal surface displacements are necessary to fit to accurate parameters.

Fitting both force and displacement data provided the best fit, and the most robust parameter set. FORCE and DISP show that multiple parameter sets can be used to fit grip force or nodal surface displacement but not both. FORCE and DISP have less precise parameter sets, with wider confidence intervals than ALL for A and B and  $\kappa$  and  $\mu$  and parameters A and B cannot be given a confidence interval for DISP due to their lack of influence on surface displacement. Thus, to characterize the tissue response accurately, parameters should be gained by ALL.

Utilizing grip forces and nodal surface displacements in the objective function for the regression of the non-linear fiber model produces the most accurate fit. We have externally validated the non-linear model that it represents real world data by determining parameter sensitivities and characterizing the response of the iliofemoral ligament.

## References

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